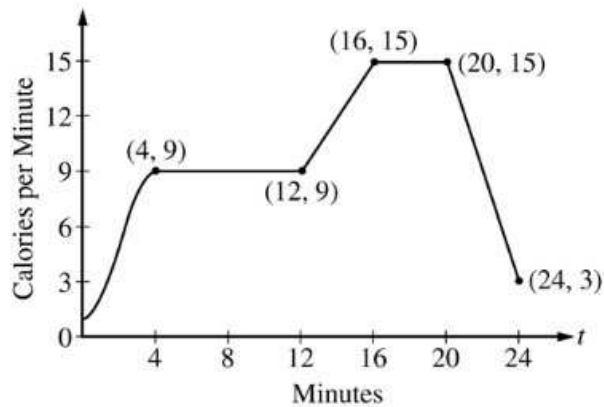


## Advanced Placement (AP) Calculus Question #1



The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function  $f$ . In the figure above,  $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$  for  $0 \leq t \leq 4$  and  $f$  is piecewise linear for  $4 \leq t \leq 24$ .

- Find  $f'(22)$ . Indicate units of measure.
- For the time interval  $0 \leq t \leq 24$ , at what time  $t$  is  $f$  increasing at its greatest rate? Show the reasoning that supports your answer.
- Find the total number of calories burned over the time interval  $6 \leq t \leq 18$  minutes.
- The setting on the machine is now changed so that the person burns  $f(t) + c$  calories per minute. For this setting, find  $c$  so that an average of 15 calories per minute is burned during the time interval  $6 \leq t \leq 18$ .

## AP Calculus Question #2

A water tank at Camp Newton holds 1200 gallons of water at time  $t = 0$ . During the time interval  $0 \leq t \leq 18$  hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

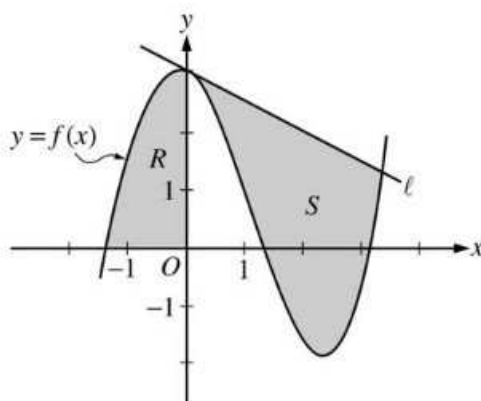
- Is the amount of water in the tank increasing at time  $t = 15$ ? Why or why not?
- To the nearest whole number, how many gallons of water are in the tank at time  $t = 18$ ?
- At what time  $t$ , for  $0 \leq t \leq 18$ , is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.
- For  $t > 18$ , no water is pumped into the tank, but water continues to be removed at the rate  $R(t)$  until the tank becomes empty. Let  $k$  be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of  $k$ .

### AP Calculus Question #3

Let  $f$  be the function defined by  $f(x) = k\sqrt{x} - \ln x$  for  $x > 0$ , where  $k$  is a positive constant.

- (a) Find  $f'(x)$  and  $f''(x)$ .
- (b) For what value of the constant  $k$  does  $f$  have a critical point at  $x = 1$ ? For this value of  $k$ , determine whether  $f$  has a relative minimum, relative maximum, or neither at  $x = 1$ . Justify your answer.
- (c) For a certain value of the constant  $k$ , the graph of  $f$  has a point of inflection on the  $x$ -axis. Find this value of  $k$ .

### AP Calculus Question #4



Let  $f$  be the function given by  $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$ . Let  $R$  be the shaded region in the second quadrant bounded by the graph of  $f$ , and let  $S$  be the shaded region bounded by the graph of  $f$  and line  $\ell$ , the line tangent to the graph of  $f$  at  $x = 0$ , as shown above.

- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -2$ .
- (c) Write, but do not evaluate, an integral expression that can be used to find the area of  $S$ .